# MAHIDOL UNIVERSITY <br> Wisdom of the Land 

## Sample Size Estimation

# \& <br> Randomization 

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## Population \& Sample


"Just so you know for next time, when we do a biopsy vie only take a tioy piece."

## Sample vs. Population



## Sample vs. Population



Fig. 3. Sampling bias. An unbiased sample is representative of and has the same characteristics as the population from which it has been drawn. A biased sample is not representative of the target population because its characteristics have different distribution as compared with the original population.

## Sample Specification

## - Inclusion Criteria



> Specifying the characteristics that define populations that are relevant to the research question and efficient for the study:

Demographic characteristics
Clinical characteristics

- Geographic (administrative) characteristics
- Exclusion Criteria
- Temporal characteristics

Specifying subset of the population that will not be studied because of:

- high likelihood of being lost to follow-up
- unable to provide good/complete data
- ethical barriers

- subject's refusal to participate


## Methods of Sampling

- Probability Sampling -- methods that utilizes some form of random selection

1. Simple Random Sampling
2. Stratified Random Sampling
3. Systematic Random Sampling
4. Cluster (Area) Sampling
5. Multi-stage Sampling

## Chance

EPS: Equal Probability of Selection

PPS: Proportionate to Size

- Non-probability Sampling - methods that based on either accidental or purposive; usually approach the sampling problem with a specific plan in mind.

1. Accidental Sampling
2. Purposive Sampling
2.1 Expert Sampling
2.2 Quota Sampling
2.3 Heterogeneity Sampling

## Relevancy

Representativeness
Specific Characteristics

## Sample Size Estimation



## Important questions in sample size estimation

What is the key outcome of interest which is to be evaluated statistically?
Cured/Not Cured, BP, Glucose conc., Survival time, No. of E. coli, ... How will the key outcome be measured?

Rate, Percent, Prevalence, Incidence, Mean, Median, etc. What kind of study does one have?
Descriptive (Parameter estimation), Analytic (Hypothesis testing)
Are there explicit or implicit dependencies in the data which need to be accounted for?
Completeness, Non-responses, Follow-up rate, Fixed /Limited sample size, Screening etc.

## Factors in sample size estimation

- A priori information about parameters of interest
- Effect size
- Confidence level (in parameter estimation) / Tail of the test (in hypothesis testing)
- Type I error ( $\alpha$, in parameter estimation) / Type I ( $\alpha$ ) \& Type II ( $\beta$ ) errors (in hypothesis testing)


## Factors in sample size estimation

- A priori information about parameters of interest
- Literature Review

From previous report, it was shown that cure rate of Drug $A=70 \%$

- Pilot Study

A pilot survey from 30 bottles of drinking water in the market shows that there are E.coli in 5 bottles.

- Expert Opinion

3 out of 5 experts say that about $10 \%$ of workers in the XXX factory have health problem related to toxic chemicals.

- Example of a priori information about parameters of interest


## Definition of Pimary Outcome: PID

- Tenderness: abdominal direct, motion of cervix and uterus, and adnexal
- GC+ or fever $>38^{\circ} \mathrm{C}$ or leucocytosis $>\mathbf{1 0 , 0 0 0} \mathbf{W B C} / \mu \mathrm{l}$ or purulent material from peritoneal cavity on culdocentesis or pelvix abscess or inflammatory complex on bimanual exam

Estimating the Incidence of PID for Sample Size Calculations

- Government officials estimated $40 \%$
- Ob/GYN from Med School estimated $\mathbf{1 2 \%}$
- Pilot study found 4\%
- We conservatively set initially at 6\%


## Factors in sample size estimation

- Effect size
- Clinical/ Public Health Importance
- Not Statistical Significance


## - Examples:

Current cure rate $=70 \%$
New drug should be $10 \%$ better => 80\%

## Previous survey found infected rate $=15 \%$

New survey expected to find infected rate not different from previous survey at $\pm 3 \%=>12-18 \%$

# Relationship Between Priori Info and Effect Size 

- Sample size is function of the
- a type I error allowed
- $\beta$ type II error allowed
- actual predicted risk
- expected reduction of risk
- The estimated sample size of each arm of a clinical trial, if the tolerated a type I error is 0.05 and $\beta$ type II error is 0.1 ?

|  | Predicted Risk |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $10 \%$ |  |
| $10 \%$ risk <br> reduction | 197,750 | 97,924 | 64,649 | 48,011 | 18,064 |  |
| $50 \%$ risk <br> reduction | 6,253 | 3,100 | 2,049 | 1,524 | 578 | $10 \%$ <br> $-1 \%$ <br> $9 \%$ |

- Type I \& Type II errors

Ho: G1 = G2
Reality/Truth


## Factors in sample size estimation

## - Type I \& Type II errors

## The Decision Matrix on Trial

## The OJ Simpson Trial Analogy



## Factors in sample size estimation

- Type I \& Type II errors



## Factors in sam Research Study

## Type of Research Study

- Descriptive
(Parameter Estimation)
- Analytic
(Hypothesis Testing)
- Research Design
- Experimental
- True Experimental (e.g. RCT)
- Quasi Experimental
- Observational
- Cross-sectional

- Case-Control
- Cohort


## Examples:

## Sample Size Estimation

- Parameter Estimation
- Hypothesis Testing




## Sample size for Parameter Estimation



Categorical outcome: $\pi=p \pm Z_{\alpha / 2} \sqrt{p(1-p) / n}$
Continuous outcome: $\mu=\bar{x} \pm Z_{\alpha / 2} \sigma / \sqrt{n}$

## Determinants of sample size (Parameter Estimation)

Three factors determine the required sample size:

1. Standard deviation, $\sigma$ (continuous); or, the proportion or prevalence rate of the outcome of interest, p (categorical).
2. The difference of the estimate that we wish to detect, $\delta$.
3. The confidence interval level (usually set at $95 \%$ CI ).

Formula:
Categorical outcome:

$$
\mathrm{Z}_{\alpha / 2}=\mathrm{Z}_{0.05 / 2} \quad \text { Priori Info. } \quad \text { Effect Size }
$$

$$
\text { Continuous outcome: } n=(1,96 \sigma / \delta)^{2}
$$

Target organ damage and cardiovascular complications in patients with hypertension and type 2 diabetes in Spain: a cross-sectional study
Luis Cea-Calvo*1, Pedro Conthe ${ }^{2}$, Pablo Gómez-Fernández³ , Fernando de Alvaro ${ }^{4}$, Cristina Fernández-Pérez ${ }^{5}$ and RICARHD investigators ${ }^{6}$

Categorical outcome
(prevalence - $p$ )

Background: Target organ damage (mainly cardiac and renal damage) is easy to evaluate in outpatient clinics and offers valuable information about patient's cardiovascular risk. The purpose of this study was to evaluate, using simple methods, the prevalence of cardiac and renal damage and its relationship to the presence of established cardiovascular disease (CVD), in patients with hypertension (HT) and type 2 diabetes mellitus (DM).

## Statistical analysis $\quad n=1.96^{2} \pi(1-\pi) / \delta^{2}$

The sample size was calculated according to the main objective of the study and based on the expected prevalence of heart and kidney damage. For an expected prevalence of ${ }^{\pi}<10 \%$, a sample size of 2401 hypertensive diabetic patients was estimated for a $95 \%$ confidence interval (CI) and an error of $\delta^{1.2 \%}$. The sample was increased $4 \%$ to cover data losses, yielding a definitive size of 2500 patients.

Cross-sectional survey on hantavirus seroprevalence in Canton St. Gallen, Switzerland

Categorical outcome
(prevalence - p)

Background and objectives: In 2002 the first endemic hantavirus infection in Switzerland was detected only by chance following a broad spectrum of diagnostics. This raised the question, whether Hantavirus infection should be included in the differential diagnosis of febrile illness of patients in Switzerland. In order to estimate the frequency of hantavirus infections in Switzerland, this survey on hantaviral seroprevalence was conducted in the Canton St. Gallen.

Cross-sectional survey on hantavirus seroprevalence in Canton St. Gallen, Switzerland

$$
n=1.96^{2} \pi(1-\pi) / \delta^{2}
$$

The sample size calculation was based on a comparison of exposure to hantaviruses measured by seroprevalence in different populations. Based on known seroprevalences in other Central European countries [22-27] an exposure of $5 \%$ in the populations with higher risk of exposure to hantaviruses and of $0.5 \%$ in blood donors was assumed. With a power of 0.8 , an alpha value of 0.05 , and a ratio of seropositive samples for the higher risk group to the blood donors of 10:1, (4) rolunteers from each population and 240 blood donors would have been needed [28].

Finally, a total of 1710 sera from 1029 blood donors, 382 farmers, 104 forestry workers, 104 soldiers, and 91 hunters were collected.

## Example: Descriptive study

## Effect size $=$ Relative precision

Suppose we are trying to estimate the prevalence of a certain disease, which we suspect to be about $3 \%$, and
We want the $95 \%$ confidence interval of the estimate to be $0.3 \%$ ( $10 \%$ of $3 \%$ ) on either side

$$
z_{\alpha / 2}=z_{0.05 / 2}
$$

Priori Info.

$$
n=1.96^{2} \pi(1-\pi) / \delta^{2}
$$

$$
n=1.96^{2} \times 0.03(1-0.03) / 0.003^{2}=12421
$$

i.e. 12421 subjects required!

## Example: Descriptive study <br> Effect size $=$ Absolute precision

A survey is being planned to estimate the prevalence of secondary infertility amongst couples aged 20-45. The investigators expect the prevalence to be $10 \%$, and
They would like to estimate it to within $5 \%$ of the true value (with 95\% confidence).
How many couples are required?

$$
\begin{gathered}
\mathrm{z}_{\alpha / 2}=\mathrm{z}_{0.05 / 2} \\
\boldsymbol{n}=\mathbf{1 . 9 6} \\
n=1.96^{2} \times 0.1 \times 0.9 / 0.05^{2}=139
\end{gathered}
$$

## Example: Descriptive study Multiple outcomes of interest

Title: Prevalence of gonoccocal, chlam ydial, hepatitis B virus and syphillis infections among pregnant women attending Hung V uong Hospital, Ho Chi M inh City, Vietnam
Sample size:
Using EPI-Info6 (epitable) calculation for sample size \& power of single proportion
Given:

- Size of the population (pregnantwomen attending Hung Vuong Hospital) $=60 /$ day orapproximately $=22000 / y r$
- Confidence level=90\%

Thus,

- Prevalence of the Chlamydia trachomatis carriers among pregnant women expected to be = 2.5\% (Citation ?) with desired precision $\%=1 \%$; sample size $=641$
- Prevalence of Treponema pallidum carriers among pregnant women expected to be=0.5\% (Citation ? ) with desired precision $\%=0.3 \%$; sample size $=1401$
- Prevalence of HBSAg_carrier among pregnant women expected to be = $10.3 \%$ (Citation ?) with desired precision $\%$ $=1.5 \%$; sample size = 1058
- Prevalence of Gonorrhoea carriers among pregnant women expected to be=0.7\% (Citation ?) with desired precision $\%=$ $0.4 \%$; sample size = 1117

If the desired precision is acceptable, the minimum sample size for this study should be 1400

## Example: Descriptive study

## Estimate continuous outcome

## Peak Expiratory Flow Rate (PEFR)

This is a simple method of measuring airway obstruction and it will detect moderate or severe disease. The simplicity of the method is its main advantage. It is measured using a standard Wright Peak Flow Meter or mini Wright Meter. The needle must always be reset to zero before PEF is measured.
(Data from Gregg et al, BMJ, 1973)
Standard deviation of PEFR: 48 litres/min


Desired $95 \%$ confidence interval width: $\pm 20$ litres/min

$$
\begin{aligned}
& n=(1.96 \sigma / \delta)^{2} \\
& n=(1.96 \times 48 / 20)^{2} \approx 22
\end{aligned}
$$

i.e. a sample of 22 would enable us to estimate the population PEFR mean to within 20 litres/min (with $95 \%$ probability).

# Sample size for Hypothesis Testing 

## (Differences between independent groups)



Categorical outcome:
Но: $\pi 1=\pi 2$
Continuous outcome:
Ho: $\mu 1=\mu 2$

# Determinants of sample size (Hypothesis Testing) 



Fig. 1. Examples of study designs. In cross-sectional studies inputs and output are measured simultaneously and their relationship is assessed at a particular point in time. In case-control studies participants are identified based on presence or absence of the disease and the temporal direction of the inquiry is reversed (retrospective). Temporal sequences are better assessed in longitudinal cohort studies where exposure levels are measured first and participants are followed forward in time. The same occurs in randomized controlled trials (RCTs) where the assignment of the exposure is under the control of the researcher. P: Probability (or risk).

## Determinants of sample size (Hypothesis Testing)

Four factors determine the required sample size

1. Standard deviation, $\sigma$ (continuous), or the expected success rate for the control group, $\mathbf{p}_{1}$ (categorical).
2. The difference between groups that we wish to detect, $\delta$.

Note: Effect size: the size of the smallest effect that is clinically important.

- Difference between two groups $=(p 1-p 2)$ or $(\mu 1=\mu 2)$
- OR/RR of two groups
e.g., 1.5 for risk of CHD in patients with hypertension (50\% increased risk)

$$
H_{0}: R R=1.0 ; H_{1}: R R>=1.5
$$

3. The false positive error rate, or significance level of the test, $\alpha$ (usually 0.05).
4. The false negative error rate, $\beta$ (usually 0.1 or 0.2 ), more commonly expressed as ( $1-\beta$ ), the power ( 0.8 or 0.9 ). This is for a specified $\delta$ (alternative hypothesis).

## Difference between two groups

## Formula: Categorical outcome

 Но: $\pi 1=\pi 2$Type I err
Power

$$
n=\frac{\underbrace{(1-\text { II })}_{\left[z_{1-\frac{\alpha}{2}}+z_{1-\beta}^{\downarrow}\right\}^{\downarrow}\left[p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)\right]}}{\left(p_{1}-p_{2}\right)^{2} \aleph_{\text {Effect Size }}}
$$

## Where

$p_{1}=$ expected/known proportion in the control group
$p_{2}=$ expected proportion in the intervention group

$$
\left(=p_{1}+\delta\right)
$$

Formula: Continuous outcome

$$
n=\frac{2\left\{\begin{array}{c}
\text { Ho: } \mu \mathbf{1}=\mu 2 \\
\text { Type I err } \\
\begin{array}{c}
\text { Power } \\
\text { (1-Type II err) } \\
\downarrow
\end{array} \\
\delta^{2}<\text { Effect Size }
\end{array}\right.}{2\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}^{2}\right\}^{2}}
$$

## Where

$\sigma=$ expected/known standard deviation in the control group
${ }^{T M}=$ expected different
standard deviation in the intervention group

# Difference between two proportions Но: $\pi 1=\pi 2$ 

Formula: Categorical outcome
Type I err

Thus

$$
n=\frac{\left\{z_{1-\frac{\alpha}{2}}^{\text {e I err }}+\begin{array}{l}
\text { Power } \\
(1-T y p e ~ I I ~ e r r) ~
\end{array}\right\}^{2}\left[p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)\right]}{\left(p_{1}-p_{2}\right)^{2}} \begin{array}{ll}
\text { Where } \\
p_{l}=\text { expect Size } & \text { in the conted propo } \text { group }
\end{array}
$$

$p_{2}=$ expected proportion in the intervention group

$$
n=\frac{7.849 \times\left[p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)\right]}{\left(p_{1}-p_{2}\right)^{2}} \quad\left(=p_{1}+\delta\right)
$$

More accurate formula:

$$
n=\frac{\left\{z_{1-\frac{\alpha}{2}} \sqrt{2 \bar{p}(1-\bar{p})}+z_{1-\beta} \sqrt{p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)}\right\}^{2}}{\left(p_{1}-p_{2}\right)^{2}}
$$

## Example: Analytic study

Intermittent versus continuous renal replacement therapy for acute kidney injury patients admitted to the intensive care unit: results of a randomized clinical trial

Robert L. Lins ${ }^{1}$, Monique M. Elseviers ${ }^{2}$, Patricia Van der Niepen ${ }^{3}$, Eric Hoste ${ }^{4}$, Manu L. Malbrain ${ }^{5}$, Pierre Damas ${ }^{6}$ and Jacques Devriendt ${ }^{7}$ for the SHARF investigators

Categorical outcome

In the SHARF 4 study, we compared prospectively the outcome of different modes of therapy [daily intermittent renal replacement therapy (IRRT) versus continuous renal replacement therapy (CRRT)]. The SHARF score was used to control for disease severity. This article will focus on the comparison between both treatment options in a randomized clinical trial with the short-term outcome on hospital mortality and renal recovery at hospital discharge used as end-points.

Intermittent versus continuous renal replacement therapy for acute kidney injury patients admitted to the intensive care unit: results of a randomized clinical trial

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$$
n=\frac{\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}\right\}^{2}\left[p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)\right]}{\left(p_{1}-p_{2}\right)^{2}}
$$

## Sample size calculation

The sample size calculation was based on the assumption that the overall mortality would $\mathrm{bP}^{15} 50^{\circ}$ as in the former SHARF studies $[19,20]$ and that a difference p2 $10 \%$ in mortality between IRRT and CRRT had to be detected to be clinically relevant. With a first-order error of 5\% and a power of $80 \%$ a sample size of 407 patients was needed in each treatment group.

Single-Dose Rufloxacin versus 3-Day Norfloxacin Treatment of Uncomplicated Cystitis: Clinical Evaluation and Pharmacodynamic Considerations

The purpose of this study was to evaluate the efficacy and safety of a single dose of rufloxacin ( 400 mg ), compared with those of a norfloxacin standard treatment ( 400 mg twice a day [BID] for 3 days).

Sample size and statistical analysis. The sample size was calculated on the basis of the expected norfloxacin cure rate of $96 \%$ (28) and a delta value of $10 \%$ to show no differences between treatments (22). With type I and II errors of 0.05 and 0.2 , respectively, 61 evaluable patients per treatment should be recruited (22). Sample size was, however, extended to 150 evaluable patients, following the IDSA/FDA guidelines for the clinical evaluation of anti-infective drug products (29).

Treatment group comparisons were done with chi-square tests and Fisher's exact test for qualitative parameters and Mann-Whitney and Spearman correlation for quantitative parameters. Two-tailed $P$ values of $<0.05$ were considered statistically significant.

Clinical efficacy of basic fibroblast growth factor

## Categorical outcome

 (bFGF) for diabetic ulcerIn the present trial, we compared the efficacy of two doses of bFGF ( $0.001 \%$ and $0.01 \%$ ) with a placebo in patients with diabetic ulcers.

The sample size was determined based on the Grade 1 achievement rate. In a 12 -week preliminary trial conducted using $0.01 \% \mathrm{bFGF}$ in patients with diabetic ulcers (data unpublished), it was observed that 5 of 8 patients achieved Grade 1 after 8 weeks' administration. Assuming that the Grade 1 achievement rate in the placebo group and $0.01 \%$ bFGF group were $30 \%$ and $60 \%$, respectively, the planned sample size of 50 patients on each group would provide a power of $80 \%$ to detect a percent difference of $30 \%$ at a significant level of 0.05 . If, however, the percent difference in the Grade 1 achievement rate was $25 \%$, the power would decrease to $64 \%$. We set the minimum number of patients which could have been shown the statistical significant difference because this study was positioned as an exploratory study.

[^0]
# Example: Analytic study RCT- Placebo vs. Treatment 

In a randomised clinical trial, the placebo response is anticipated to be $25 \%$, and the active treatment response 65\%.
How many patients are needed if a two-sided test at the $5 \%$ level is planned, and a power of $80 \%$ is required?

$$
\begin{aligned}
& n=\frac{7.849 \times\left[p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)\right]}{\left(p_{1}-p_{2}\right)^{2}} \\
& n=\frac{7.849 \times(0.25(1-0.25)+0.65(1-0.65))}{(0.25-0.65)^{2}}
\end{aligned}
$$

$$
=\frac{7.849 \times 0.415}{0.16}=20.4
$$

$$
\text { so } \mathrm{n}=21 \text { per group. }
$$



## Example: Analytic study Case-Control Design

Does oral contraceptive use cause myocardial infarction?
$p_{1}=$ proportion of exposure among controls $=10 \%=0.10$
$p_{2}=$ proportion of exposure among cases $=\left(p_{1}\right)(\mathrm{PR})$
$=(0.10)(1.8)=0.18$ if assume Prev Ratio $=1.8$
$q_{2}=1-p_{2}=1-0.18=0.82$
$q_{1}=1-p_{1}=1-0.10=0.90$
$z_{1-\alpha / 2}=$ value of the standard normal distribution corresponding to alpha: e.g., 1.96 for 2 -sided test at 0.05
$z_{1-\beta}=$ value of the standard normal distribution corresponding to desired power level: e.g., 0.84 for a power of $80 \%$

Outcome
(Case)


$$
\mathrm{n}(\text { each group })=\frac{[(0.1)(0.9)+(0.18)(0.82)][1.96+0.84]^{2}}{(0.18-0.10)^{2}}=\frac{(0.2376)(7.84)}{0.0064}=291.06
$$

# Example: Analytic study Case-Control Design 

Table for $\mathbf{N}$ for OC and MI Study

| Postulated Prevalence <br> Ratio | Required n/group |
| :--- | :--- |
| 1.2 | 3834 |
| 1.3 | 1769 |
| 1.5 | 682 |
| 1.8 | 291 |
| 2.0 | 196 |
| 2.5 | 97 |
| 3.0 | 59 |

Assumes 10\% use of OC in population, power=80\%, alpha=0.05

## Но: $\mu 1=\mu 2$

Formula: Continuous outcome

$$
n=\frac{2\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}\right\}^{2} \sigma^{2}}{\delta^{2} \text { Eype I effect Size }}
$$

The most commonly used value for significance $(\alpha)$ is 0.05 , giving $z_{1-\alpha / 2}=1.96$

The most commonly used value for power (1- $\beta$ ) is $80 \%$, giving
$z_{1-\beta}=0.84$

$$
n=\frac{2\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}\right\}^{2} \sigma^{2}}{\delta^{2}}=\frac{2\{1.96+0.84\}^{2} \sigma^{2}}{\delta^{2}}=\frac{2 \times 7.849 \sigma^{2}}{\delta^{2}} \approx \frac{15.7 \sigma^{2}}{\delta^{2}}
$$

## Example: Analytic study

## Continuous outcome

## Proof-Of-Concept Trial In Patients With Dominant Polycystic Kidney Disease:

## Study Aim

The primary objective of the SUISSE ADPKD study is to assess the effectiveness of sirolimus to retard kidney volume growth and to prevent the loss of renal function in young patients with ADPKD and preserved renal function. Patients with ADPKD and kidney volume growth that can be documented within 6 months are randomized to treatment with sirolimus $2 \mathrm{mg} /$ day for 18 months (Figure 1) or standard treatment. The secondary objectives are to follow renal function and blood pressure and to monitor for the occurrence of proteinuria. Safety and tolerability of sirolimus treatment in ADPKD patients will also be assessed.

## Example: Analytic study

## Proof-Of-Concept Trial In Patients With Dominant Polycystic Kidney Disease:

$$
n=\frac{2\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}\right\}^{2} \sigma^{2}}{\delta^{2}}
$$

## Sample Size Considerations

In a large cohort of ADPKD patients, the mean annual kidney volume growth rate was $5.27 \% \pm 3.92 \%$ (SD).[7] Because patients with lack of progression during the pre-randomisation period will be excluded from our study, we expect to select for a higher progression rate in our study population. Due to a shorter observation interval compared to the mentioned observational study, the standard deviation might be higher. Presuming an annual kidney growth rate of $6 \% \pm 4.75 \%$ (SD) D the control group, a sample size of 40 patients per group will have $80 \%$ statistical power to detect a $50 \%$ relative reduction of kidney volume growth using a two-sided a-level of 0.05 . To account for a drop out rate of up to $20 \%$, we plan to randomise a total of 100 patients.

## Example: Analytic study RCT - Continuous outcomes

In a trial to compare the effects of two oral contraceptives on blood pressure (over one year), it is anticipated that one drug will increase diastolic blood pressure by 3 mmHg , and the other will not change it. The SD (of the changes in blood pressure) in both groups is expected to be 10 mmHg .
How many patients are required for this difference to be significant at the $5 \%$ level (with $80 \%$ power)?

$$
\begin{aligned}
& n=\frac{2\left\{z_{1-\frac{\alpha}{2}}+z_{1-\beta}\right\}^{2} \sigma^{2}}{\delta^{2}}=\frac{2\{1.96+0.84\}^{2} \sigma^{2}}{\delta^{2}}=\frac{2 \times 7.849 \sigma^{2}}{\delta^{2}} \\
& n=\frac{2 \times 7.849 \times 100}{9}=175 \\
& \text { So n }=175 \text { women per group }
\end{aligned}
$$





## Cross-over Design



Source: Deborah Grady, Introduction to Randomized Clinical Trials

## Factorial Design



|  | Int A | Plb A |
| :--- | :---: | :---: |
|  |  |  |
| Int B | B |  |
| Plb B | A | Pbo |
|  |  |  |

Int A Int B


## Table 1. Hypotheses Associated with the Different Types of Studies when Comparing a New Therapy Against a Current Therapy with Respect to Efficacy

| Type of study | Null hypotheses | Research hypothesis |
| :--- | :--- | :--- |
| Traditional <br> comparative | There is no difference <br> between the therapies | There is a difference <br> between the therapies |
| Equivalence | The therapies are <br> not equivalent | The new therapy is <br> equivalent to <br> current therapy |
| Noninferiority | The new therapy is inferior <br> to the current therapy | The new therapy is <br> not inferior to <br> the current therapy |

[^1][^2]Efficacy is measured by success rates, where higher is better.


Efficacy is measured by failure rates, where lower is better.



Figure 1. Two one-sided test procedure (TOST) and the equivalence margin in equivalence/noninferiority testing.

## Source: E Walker \& A S. Nowacki, Understanding Equivalence and Noninferiority Testing

[^3]2 Comparison of superiority, equivalence and noninferiority* hypotheses based on a $2 \%$ margin of difference in event rates

$H_{0}=$ null hypothesis. $H_{a}=$ alternative hypothesis.
$\Delta=$ difference in event rates between new and standard treatments.
$S=\Delta$ in superiority trials. $E=\Delta$ in equivalence trials.
NI $=\Delta$ in non-inferiority trials.

* Testing for non-inferiority is in one direction only-even if superiority exists (dashed arrow), it is not the hypothesis being tested.


## Equivalence / Non-inferior Trials Blephics Equivalence / Non-inferior Trials ${ }^{8}$

2 Comparison of superiority, equivalence and noninferiority* hypotheses based on a $2 \%$ margin of difference in event rates

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Source: Ian A Scott, MJA • Volume 190 Number 6 • 16 March 2009

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NI $=\Delta$ in non-inferiority trials.

* Testing for non-inferiority is in one direction only-even if superiority exists (dashed arrow), it is not the hypothesis being tested.

Source: Ian A Scott, MJA • Volume 190 Number 6 • 16 March 2009

# New Approaches: Bayesian / Adaptive Designs 

Figure 11. Paradigm Shift

|  | Old | New |
| :--- | :--- | :--- |
| Inferential Process | Hypothesis Testing <br> (Attempt to reject null hypothesis) | Continuous Learning <br> (Update probabilities of alternative hypotheses) |
| Question Being Addressed | How likely are the trial results, given there really <br> is no difference among treatments? | How likely is there a true difference among <br> treatments, given the trial data? |
| Drug Approval | Pivotal Trial <br> Distinct Phase 0-IV Trials | Weight of Evidence <br> Continuous Trials |
| Trial Designs | Single Stage | Adaptive |
| Statistics | Traditional | Bayesian |



## Sample Size Formula for Variations of CT - Continuous Outcomes

| รูปแบบการวิจัย | สมมติฐาน | สมมติฐานและสูตรคำนวณขนาดกสู่งตัวอย่าง |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $H_{O}$ | $H_{a}$ | สูตรพื้นฐาน |
| กลุ่มเดี่ยว <br> (One - Sample) | Equality | $\mu-\mu_{0}=0$ | $\mu-\mu \mu_{0} \neq 0$ | $n=\frac{\left(z_{\frac{\alpha}{2}}+z_{\mathcal{p}}\right)^{2} \sigma^{2}}{\left(\mu-\mu_{0}\right)^{2}}$ |
|  | Superiority | $\mu-\mu_{0} \leq \mathcal{S}$ | $\mu-\mu_{0}>\mathcal{S}$ | $n=\frac{\left(z_{\alpha}+z_{\mathcal{B}}\right)^{2} \sigma^{2}}{\left(\mu-\mu_{0}-\delta\right)^{2}}$ |
|  | Equivalence | $\left\|\mu-\mu_{0}\right\| \geq \delta$ | $\left\|\mu-\mu_{0}\right\|<\delta$ | $n=\frac{\left(z_{\alpha}+z_{\mathcal{B}}\right)^{2} \sigma^{2}}{\left(\left\|\mu-\mu \mu_{0}\right\|-\delta\right)^{2}}$ |
| สองกลุ่มคู่ขนาน <br> (Two - sample <br> Parallel) | Equality | $\mu_{1}-\mu_{2}=0$ | $\mu \mu_{1}-\mu_{2} \neq 0$ | $n_{i}=\frac{2\left(z_{\frac{\alpha}{2}}+z_{\beta}\right)^{2} \sigma^{2}}{\left(\mu_{1}-\mu_{2}\right)^{2}}$ |
|  | Non-inferiority | $\mu_{1}-\mu_{2} \geq \mathcal{S}$ | $\mu \mu_{1}-\mu_{2}<\mathcal{S}$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\beta}\right)^{2} \sigma^{2}}{\left(\mu_{1}-\mu_{2}-\delta\right)^{2}}$ |
|  | Superiority | $\mu_{1}-\mu_{2} \leq \mathcal{S}$ | $\mu_{1}-\mu_{2}>\delta$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\mathcal{B}}\right)^{2} \sigma^{2}}{\left(\mu_{1}-\mu_{2}-\delta\right)^{2}}$ |
|  | Equivalence | $\left\|\mu_{1}-\mu_{2}\right\| \geq \mathcal{S}$ | $\left\|\mu_{1}-\mu_{2}\right\|<\mathcal{S}$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\mathcal{\beta}}\right)^{2} \sigma^{2}}{\left(\left\|\mu_{1}-\mu_{2}\right\|-\delta\right)^{2}}$ |
| สองกลุ่มไขวั (Two sample Crossover) | Equality | $\mu_{1}-\mu_{2}=0$ | $\mu \mu_{1}-\mu_{2} \neq 0$ | $n_{z}=\frac{\left(z_{\frac{\alpha}{2}}+z_{\mathcal{P}}\right)^{2} \sigma^{2}}{2\left(\mu_{1}-\mu_{2}\right)^{2}}$ |
|  | Non-inferiority | $\mu_{1}-\mu_{2} \geq \delta$ | $\mu_{1}-\mu_{2}<\mathcal{S}$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{F}}\right)^{2} \sigma^{2}}{2\left(\mu \mu_{1}-\mu \mu_{2}-\delta\right)^{2}}$ |
|  | Superiority | $\mu_{1}-\mu_{2} \leq \mathcal{S}$ | $\mu_{1}-\mu_{2}>\mathcal{S}$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{F}}\right)^{2} \sigma^{2}}{2\left(\mu_{1}-\mu_{2}-\delta\right)^{2}}$ |
|  | Equivalence | $\left\|\mu_{1}-\mu_{2}\right\| \geq \delta$ | $\left\|\mu \mu_{1}-\mu_{2}\right\|<\mathcal{S}$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{A}}\right)^{2} \sigma^{2}}{2\left(\left\|\mu_{1}-\mu \mu_{2}\right\|-\delta\right)^{2}}$ |

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Sample Size Formula for Variations of CT - Categorical Outcomes

|  |  | สมมติฐานแล | คำนวณขนาดกลุ่มตัวอย่าง |
| :---: | :---: | :---: | :---: |
| รูปแบบการวิจัย | สมมติฐาน | $\mathrm{H}_{0}$ | สูตรพื้นฐาน |
| กสุ่มเตี่ยว (One - Sample) | Equality | $\pi-\pi_{0}=0$ | $n=\frac{\left(z_{\frac{\alpha}{2}}+z_{\rho}\right)^{2} \pi(1-\pi)}{\left(\pi-\pi_{0}\right)^{2}}$ |
|  | Superiority | $\pi-\pi_{0} \leq \mathcal{\delta}$ | $n=\frac{\left(z_{\alpha}+z_{\mathcal{R}}\right)^{2} \pi(1-\pi)}{\left(\pi-\pi_{0}-\mathcal{S}\right)^{2}}$ |
|  | Equivalence | $\left\|\pi-\pi_{0}\right\| \geq \delta$ | $n=\frac{\left(z_{\alpha}+z_{\mathcal{R}}\right)^{2} \pi(1-\pi)}{\left(\left\|\pi-\pi_{0}\right\|-\mathcal{S}\right)^{2}}$ |
| สองกสู่มคู่บนาน <br> (Two - sample <br> Parallel) | Equality | $\pi_{1}-\pi_{2}=0$ | $n_{i}=\frac{2\left(z_{\frac{\alpha}{2}}+z_{\beta}\right)^{2}\left(\pi_{1}\left(1-\pi_{1}\right)+\pi_{2}\left(1-\pi_{2}\right)\right)}{\left(\pi_{1}-\pi_{2}\right)^{2}}$ |
|  | Non-inferiority | $\pi_{1}-\pi_{2} \geq \mathcal{S}$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\mathcal{f}}\right)^{2}\left(\pi_{1}\left(1-\pi_{1}\right)+\pi_{2}\left(1-\pi_{2}\right)\right)}{\left(\pi_{1}-\pi_{2}-\delta\right)^{2}}$ |
|  | Superiority | $\pi_{1}-\pi_{2} \leq \mathcal{S}$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\mathcal{R}}\right)^{2}\left(\pi_{1}\left(1-\pi_{1}\right)+\pi_{2}\left(1-\pi_{2}\right)\right)}{\left(\pi_{1}-\pi_{2}-\delta\right)^{2}}$ |
| สองกลู่มไขวั <br> (Two sample Crossover) | Equivalence | $\left\|\pi_{1}-\pi_{2}\right\| \geq \delta$ | $n_{i}=\frac{2\left(z_{\alpha}+z_{\mathcal{B}}\right)^{2}\left(\pi_{1}\left(1-\pi_{1}\right)+\pi_{2}\left(1-\pi_{2}\right)\right)}{\left(\left\|\pi_{1}-\pi_{2}\right\|-\mathcal{S}\right)^{2}}$ |
|  | Equality | $\pi_{1}-\pi_{2}=0$ | $n_{i}=\frac{\left(z_{\frac{\alpha}{2}}+z_{\mathcal{A}}\right)^{2} \sigma_{d}^{2}}{2\left(\pi_{1}-\pi_{2}\right)^{2}}$ |
|  | Non-inferiority | $\pi_{1}-\pi_{2} \geq \delta$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{F}}\right)^{2} \sigma_{d}^{2}}{2\left(\pi_{1}-\pi_{2}-\delta\right)^{2}}$ |
|  | Superiority | $\pi_{1}-\pi_{2} \leq \mathcal{S}$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{F}}\right)^{2} \sigma_{d}^{2}}{2\left(\pi_{1}-\pi_{2}-\delta\right)^{2}}$ |
|  | Equivalence | $\left\|\pi_{1}-\pi_{2}\right\| \geq \delta$ | $n_{i}=\frac{\left(z_{\alpha}+z_{\mathcal{\beta} / 2}\right)^{2} \sigma_{d}^{2}}{2\left(\left\|\pi_{1}-\pi_{2}\right\|-\delta\right)^{2}}$ |

## Websites for Sample Size Calculation



1. Adjusting for loss of follow up

- If p is the proportion of subjects lost to follow-up, the number of subjects must be increased by a factor of $1 /(1-\mathrm{p})$.

$$
\text { - } N_{a d j}=\mathrm{N} \times 1 /(1-\mathrm{p})
$$

2. Adjusting for Non-adherence

- Ro =drop out rate
- Ri=drop in rate

Nadj $=N /\left(1-R_{o}-R_{I}\right)^{2}$

- If $\mathrm{Ro}=0.20, \mathrm{Ri}=0.05$

Nadj $=1.78 \mathrm{~N}$ <br> \section*{\title{
Sample Size Adjustment <br> \section*{\title{
Sample Size Adjustment <br> <br> Example:} <br> <br> Example:}

Statistics Guide for Research Grant Applicants: D. Sample size calculations - Microsoft Internet Explorer
-
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Scenario: The prevalence of dysfunctional breathing amongst asthma patients being treated in general practice is to be assessed using a postal questionnaire survey (Thomas et al. 2001).

Required information: -

- Primary outcome variable = presence/absence of dysfunctional breathing
- 'Best guess' of expected percentage (proportion) $=30 \%(0.30)$
- Desired width of $95 \%$ confidence interval $=10 \%$ (i.e. $+/-5 \%$, or $25 \%$ to $35 \%$ )

The formula for the sample size for estimation of a single proportion is as follows: -
$n=15.4^{*} p^{*}(1-p) / W^{2}$
where $n=$ the required sample size $p=$ the expected proportion - here $0.30 \mathrm{~W}=$ width of confidence interval - here 0.10

Inserting the required information into the formula gives: -
$n=15.4 * 0.30 *(0.70) / 0.10^{2}=324$

$$
324 \times 1 /(1-.3)=463
$$

Suggested description of this sample size calculation: -
"A sample of 324 patients with asthma will be required to obtain a $95 \%$ confidence interval of $+/-5 \%$ around a prevalence estimate of $30 \%$. To allow for an expected $70 \%$ response rate to the questionnaire, a total of 480 questionnaires will be delivered."

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## Sample Size Adjustment

Scenario: A randomised controlled trial has been planned to evaluate a brief psychological intervention in comparison to usual treatment in the reduction of suicidal ideation amongst patients presenting at hospital with deliberate self-poisoning. Suicidal ideation will be measured on the
importance. It is anticipated that around one third of patients may drop out of treatment (Guthrie et al. 2001)
Required information: -

- Primary outcome variable $=$ The Beck scale for suicidal ideation. A continuous variable summarised by means.
- Standard deviation $=7.7$ points

Size of difference of clinical importance $=5$ points

- Significance level $=5 \%$
- Power = 80\%
- Type of test $=$ two-sided

The formula for the sample size for comparison of 2 means (2-sided) is as follows: -
$n=[A+B]^{2} * 2^{*} S D^{2} /$ DIFFF
where $n=$ the sample size required in each group (double this for total sample)
$S D=$ standard deviation, of the primary outcome variable - here 7.7.
DIFF $=$ size of difference of clinical importance - here 5.0
A depends on desired significance level (see table) - here 1.96

| Significance level | A |
| :--- | :---: |
| 58 | 1.96 |
| 18 | 2.58 |
| POwer | $B$ |
| 808 | 0.84 |
| 908 | 1.28 |
| 958 | 1.64 |

Inserting the required information into the formula gives: -
$n=[1.96+0.84]^{2 * 2 * 7.72 / 5.0^{2}-38}$
This gives the number required in dach of the trial's two groups. Therefore the total sample sizelis double this, i.e. 76 .
To allow for the predicted dropout rate of around one third, the sample size was increased to 60 in each group, a total sample of 120
Suggested description of this sample size calculation: -
"A sample size of 38 in each group will be sufficient to detect a difference of 5 points on the Beck scale of suicidal ideation, assurning a 120), to allow for a predicted drop-out from treatment of around one third"

Back to top <br> \section*{Example: <br> \section*{Example: <br> <br> :Sample Size Adjustment} <br> <br> :Sample Size Adjustment} Wisdom of the Land

Randomised, double-blind, placebo-controlled study to determine whether steroids reduce the incidence and severity of nephropathy in Henoch Schonlein Purpura (HSP)

Main research questions

## Do steroids reduce the incidence and severity of nephropathy in childhood HSP?

Are ACE genotype polymorphisms predictive of progressive nephropathy in HSP?
Henoch-Schonlein Purpura (HSP) is the commonest small vessel vasculitis of childhood. Long term prognosis is related to progressive renal insufficiency. There is no conclusive evidence that steroids will alter the course of the disease. We will address this. In conjunction, we will evaluate the association between insertion and deletion polymorphisms of the ACE gene and progressive nephropathy in treated and untreated groups.

## Data analysis/Sample size

Formal statistical input into the study has been provided by the Research and Development Support Unit at Southmead hospital, Bristol. To test the hypothesis that treatment with prednisolone $2 \mathrm{mg} / \mathrm{Kg}$ for a period of 14 days reduces the incidence of proteinuria at a set point ( 12 months) after initial presentation. We will require a study of 320 patients ( 160 in each group). This calculation is based on the premise that $15 \%$ of children in the untreated group are likely to develop proteinuria during the 12 month period, compared with $5 \%$ in the treated group. This sample size will provide $80 \%$ power for testing the hypothesis at the $5 \%$ level of statistical significance, and assumes the difference will be analysed using a continuity corrected chi-squared test. Allowing a dropout rate of $15 \%, 184$ patients will need to be randomised to each treatment arm (prednisolone or placebo).

## Randomization Random Allocation Random Assignment



## Random allocation

- Assures subjects have same probability of being assigned to either experimental or control groups
- Has effect of increasing comparability of groups Groups similar with regard to distribution of anticipated, and unanticipated, confounders in terms of -
- all factors other than the intervention being applied
- essentially eliminates selection bias


## Randomization Rules:

- Use a procedure that really allocates randomly.
- 30-50\% of 287 RCTs did not describe an appropriate randomization procedure.
- Use a procedure that is tamperproof.
- $25 \%$ of 287 RCTs did not provide adequate concealment.


## Block Randomization

- Assures equal distribution
- Blocks of 4: randomly arrange the order of these six possible groupings.
TTCC TCTC TCCT CCTT CTCT CTTC
- Problem: easier to guess next assignment


## Example:

## Randomization Process/Coding

## Example: 3 Blocks for Part A1 (12 volunteers)

| Random <br> Number <br> Sequence | Permuted <br> Block |
| :--- | :--- |
| 1 | P L L L |
| 2 | L P L L |
| 3 | L L P L |
| 4 | L L L P |


| Randomization <br> List |  | Study ID |
| :---: | :---: | :---: |
| 3 | L | 911001 |
|  | L | 911002 |
|  | P | 91103 |
|  | L | 911004 |
| 2 | L | 911005 |
|  | P | 911006 |
|  | L | 91107 |
|  | L | 911008 |
| 4 | L | 911009 |
|  | L | 911010 |
|  | L | 911011 |
|  | P | 911012 |

Statistician \& GPO Pharmacist know the coding


## Example:

Study ID



## Random number generator program

http://www.saccenti.com/randomnumber/randomnumber.htm


## Free Random Number Generator

- Simplest thing in the world, a little 20 KB program
- Should run on any Windows, 95 to present
- Created using Microsoft Visual Basic 6.0
- Will download as "randomnumber.exe" (click here to download)
- Random Number Generator copyright 2003-2011 by Scott Donato Saccenti (http://www.saccenti.com/index.htm).
- No charge, but if you find the program useful and would like to thank me with a small donation, a couple bucks would be appreciated. I can accept donations through a credit card via Paypal. My address for receiving payments is paypal"at"saccenti.com (replace the "at" with the usual symbol...I'm trying to avoid spam by keeping the real address off this web page). Thanks.

Here is what the program looks like:

| E. Random |  | -回区 |
| :---: | :---: | :---: |
| Enter number | 488 |  |
| Get result | 229 |  |
|  | 015 |  |



[^4]
## Random number generator program

http://www.randomizer.org/


[^5]
## Sample Size Estimation \& Randomization Process



## Example: RV144 (2009)

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## Example: RV144 (2009)

Wisdom of the Land

C Modified Intention-to-Treat Analysis



Vaccination with ALVAC and AIDSVAX to Prevent HIV-1 Infection in Thailand

## ประสิทิิิิล <br> 0.279-0.192 0.279

Table 2. Rate of HIV Infection and Vaccine Efficacy, According to Selected Baseline Variables (Modified Intention-tp-Treat Population).

| Variable | Vaccine ( $\mathrm{N}=8197$ ) |  |  | Rate no./person- | No. <br> Evaluated | Placebo ( $\mathrm{N}=8198$ ) |  |  |  | accine Efficacy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. <br> Evaluated | No. with Infection | No. of PersonYears |  |  | No. with Infection | No. of PersonYears |  |  |  |
|  |  |  |  |  |  |  |  |  |  | \% (95\% CI) |
| All subjects | 7960 | 51 | 26,507 |  | 7988 | 74 | 26,478 | 0.279 | 31.2 | 1.7 to 51.8) |


[^0]:    www.biophics.org

[^1]:    Source: E Walker \& A S. Nowacki, Understanding Equivalence and Noninferiority Testing

[^2]:    www.biophics.org

[^3]:    www.biophics.org

[^4]:    www.biophics.org

[^5]:    www.biophics.org

